NBSIR 75-900
Liquid Level Instrumentation in Volume Calibration

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Institute for Basic Standards
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Sponsored in part by:
Brookhaven National Laboratory
Technical Support Organization
Upton, L.I., New York 11973

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U.S. DEPARTMENT OF COMMERCE, Elliot L. Richardson, ***Secretary***
James A. Baker, III, ***Under Secretary***
Dr. Betsy Ancker-Johnson, ***Assistant Secretary for Science and Technology***
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, ***Acting Director***

Liquid Level Instrumentation in Volume Calibrations

by

R. Schoonover, H. Ku, J. Whetstone and J. Houser
with Appendix by C.R. Tilford

Introduction

Several years ago, the Mass and Volume Section of the National Bureau of Standards undertook a study, supported in part by the Atomic Energy Commission, to determine the accuracy of a nuclear fuel reprocessing tank volumetric calibration. The calibration technique studied was volume versus liquid level as determined by head pressure measurements. The results are given in NBS Report 10396 [1].*

The purpose of this study is two-fold: (1) to use the earlier work as a basis for comparison of several instruments as liquid level detectors including the appropriate analysis and (2) to generate data for testing the ANSI Draft Standard N15.19. The proposed ANSI standard sets forth an analysis technique for application to tank calibration data.

Data reduction is covered in the previous report and is supplemented by sight glass observations, liquid density measurements, and more general equations in the appendices of this report. Included are the data necessary for an independent evaluation and analysis.

Review

This study relies on earlier work described in NBS Report 10396 for calibration techniques and data reduction. A short review of that work is presented here.

That report detailed the calibration and data reduction of straight-wall tanks via liquid level head pressure. To measure head pressure, a quartz bourdon gage with an electro-mechanical readout was used. This type of instrument is sufficiently sensitive and reproducible, but requires calibration to obtain suitable accuracy.

With the proper pressure gage coupled to a tank, there must be some means of introducing known volume increments and noting the resulting head pressure. There are two popular methods of introducing known volume increments. Both are generally performed using water during initial tank calibrations. One

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* The numbers in brackets refer to similarly numbered references at the end of this paper.

method is to pour water from a standard test measure of known volume and the other is to weigh out a volume of water and then pour it into the tank. The second method can be modified by first pouring an unknown volume increment into the tank and then weighing the tank to determine the added volume.

Although the two methods for determining incremental volumes were found comparable [2], the method of volume transfer is more convenient. The calibration results in the correspondence of liquid level (height) to tank volume. The uncertainty of the correspondence is about 3 parts in 10^4.

In proceeding with the current study, it was necessary to replace the original pressure probe (bubbler tube) and make other modifications to the tank. These modifications preclude comparing the tank's geometric profile to the previous one.

Current Study

As stated above, an overall tank calibration accurate to 3 parts in 10^4 is obtainable if proper care is exercised. Our goal here is to compare three pressure gages and a sight glass as liquid level detectors, via tank calibration, suitable for field operation at the 1 part in 10^3 level. Included in these measurements is a test for maximum level detection resolution. Additionally, the water density is determined by direct pressure measurement and also calculated based on temperature measurements.

NBS built the sight glass detector and supplied the XR-38 pressure gage. This gage was used in the previous study. The Brookhaven Technical Support Organization (TSO) arranged for two other gages, a Ruska DDR 6000 and a Bell and Howell electromanometer, model number 4-336-0050, with servo amplifier 11-169.[1]

Although we did not require an experiment at the 3 parts in 10^4 level of accuracy, the XR-38 pressure gage and tank calibration techniques were used as before. Thus, we had a standard of known performance from which to judge the other detectors and the techniques used in conjunction with them.

We used volumetric transfer from a standard test measure accurate to about 3 parts in 10^5 as the means of providing known volume increments for the tank calibration. A gravimetric check was provided in conjunction with the first calibration test merely to verify that the system behaved as expected.

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[1] Certain commercial equipment, instruments, or materials are identified in this paper in order to adequately specify the experimental procedure. In no case does such identification imply recommendation or endorsement by the National Bureau of Standards, nor does it imply that the material or equipment identified is necessarily the best available for the purpose.

It is noteworthy to stress that the calibration of a processing tank as presented here is a system calibration valid under certain restrictions. These restrictions are discussed in appendix 3.

Hardware

The steel tank was 4 feet in diameter and 14 feet high, but was modified to accommodate three sets of probes. Each set consisted of two bubbler tubes, the orifice separation of which was 20 cm. Support for the probes is furnished by three horizontal braces that span the tank's inner diameter. Over the working range of the tank, these braces cause discontinuities in the right-circular cylindrical geometry. Placement of the bubbler orifices near the tank's bottom end provides the maximum detection range and minimizes the heel volume of the tank. The tank and heel volumes are, as before, 3000 liters and 200 liters respectively.

A sight glass of 25 mm bore is in direct contact with a measuring tape and is mounted vertically to the tank's exterior wall. At the bottom end, a connection is made between the glass tube and the tank's contents. The upper tube end is in free contact with the atmosphere. The tape itself is three meters in length and has one millimeter graduations. The uniformity and correctness of the graduations has been verified by independent calibration [3].

Dry nitrogen is used for the bubbler gas source and is regulated at one cubic foot per hour regardless of head pressure for both the DDR 6000 and Bell and Howell gages. These two gages both work in much the same manner. The pressure is sensed by a quartz bourdon tube in the DDR 6000 and a metal bellows in the Bell and Howell instrument. In each instrument the elastic sensing element is servoed to a null position. The restoring force is supplied and controlled electronically in each instrument and requires a five and one-half digit (5½) digital voltmeter (DVM) for output of the pressure signal. One DVM was used for reading both instruments.

The DDR 6000 was factory adjusted and calibrated to read out directly in cm of water at 0 °C with the DVM set on the 10 VDC voltage range. The manufacturer specifies linearity to be 0.002% full scale or better with a pressure range of 10 psiG. The Bell and Howell instrument calibration is 1 volt DC = 1 psi with a range of 5 psi. Early experimentation indicated the voltage outputs for these instruments required integration to produce a steady DVM display. This was provided by an external RC filter network with a one-second time constant.

To simulate field use of these instruments, they were not pressure cycled over the expected working range to minimize hysteresis. Although unproven here, null operation probably eliminates the advantage of an exercise cycle.

Other than the modifications specifically mentioned, the tank configuration remains the same as that of reference 1. Shown in figure 1 is the experimental setup used for this study.

Design and Analysis of Eight Calibration Runs

Four criteria largely determined the number of repeated calibrations and their composition: first, TSO desired each calibration to contain at least 17 points per run for testing the proposed ANSI Standard N15.19; second, one calibration run should test the resolution of the various level detectors; third, from a statistical viewpoint, each calibration should not contain nominally identical data points; fourth, the tank should be weighed empty and full in order to demonstrate closure of the gravimetric and volumetric calibration methods.

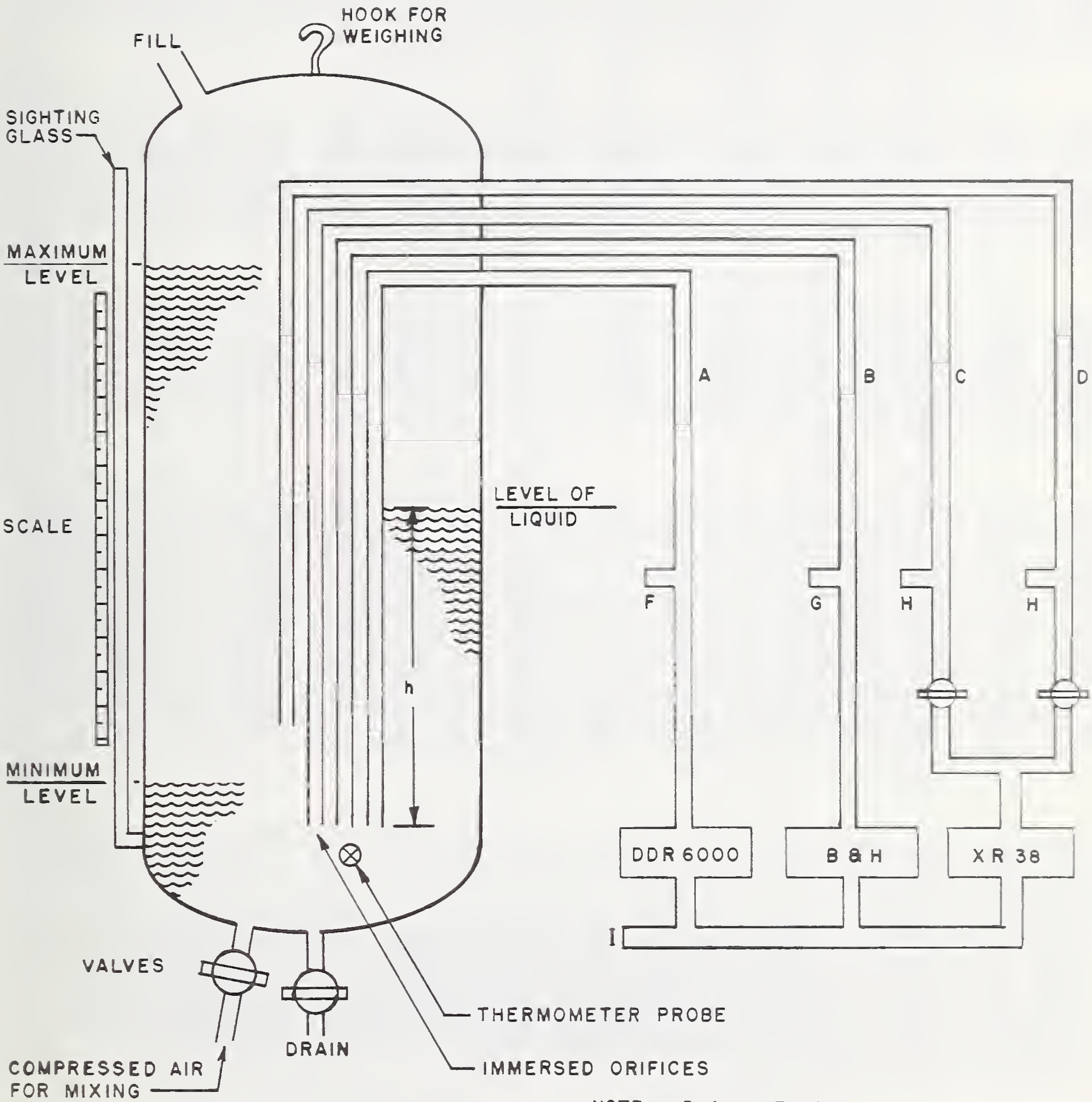
We made eight calibration measurements, most of which have some volume variation from each other. Table 1 gives the volume increment for each run in the order of occurrence. Our first measurement tested the integrity of the bubbler tubes. This was accomplished by measuring the same liquid level with the XR 38 connected individually to each probe. However, pressure was not measured for every increment of the run which results in fewer calibration points. A gravimetric check of total accumulated volume was done on Run II and was in agreement with the volumetric technique within the uncertainties of the two. Run IV contains many increments ordered in a manner to test detector resolution.

Except for the additional bubbler tube needed for measuring water density and the sight glass observation, data reduction is the same as that of NBS Report 10396. (See the appendices of this report for details concerning data reduction for the sight glass and water density measurement). The calibration results are tabulated for each detector in reduced form (see table 2). Each point is standardized to 20 °C and is adjusted for variations in bubbler depths. That is, the first point, or instrument indication, is subtracted from all others observed with the instrument to provide a common intercept (volume at zero height). Temperature corrections on the sight glass data have been made for runs 6, 7, and 8, but not for runs 1 through 5 because we did not observe sight glass temperature. A calibration line was fitted by the method of least squares to data from each of the four instruments within a run using the following model:

$$h = a + b\,V + E + e$$

where V: The volume of water added, corrected for temperature and density, in liters;

h: The observed height of the liquid level as indicated by the particular instrument in cm;



NOTE: Points F, G, and H are connected to pressure source and I is open to the atmosphere.

FIGURE 1: SIMPLIFIED VIEW OF HARDWARE HOOKUP

Calibration Runs

	#1	#2	#3	#4	#5	#6	#7	#8
1	[50]	50	100	100	50	55	65	30
2	50	100	100	100	50	50	50	50
3	[50]	100	100	100	50	50	50	50
4	50	100	100	1	50	50	50	50
5	[50]	100	100	1	50	50	50	50
6	50	100	100	5	50	50	50	50
7	[50]	100	100	5	50	50	50	50
8	50	100	100	1	50	50	50	50
9	[50]	100	50	1	50	50	50	50
10	50			1/4	50	50	50	50
11	[50]			1/2	50	50	50	50
12	50			3/4	50	50	50	50
13	[50]			1	50	50	50	50
14	[50]			100	50	50	.50	50
15	[50]			100	50	50	50	50
16	[50]			100	50	50	50	50
17	50			100	50	50	50	50
18				100				
19				100				

Table 1. Shown above are the nominal volume increments for each calibration in order of occurrence. For clarity, the nominal volumes are given in gallons and elsewhere in the text as liters. Brackets indicate the omission of the pressure observation.

RUN 1

TR NO.	TANK TEMP.	WATER DENSITY	WATER MASS	ACCUM. VOLUME	DELTA VOLUME	SIGHT GLASS HEIGHT	DELTA HEIGHT	PROBE 1 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT	PROBE 1 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT	PROBE 1 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT
2	14.246	999.208	378.35	.37869	.00000	.00000*	.00000	2435.3	.24897	.00000	2488.5	.25441	.00000	2452.0	.25067	.00000
4	14.110	999.227	756.39	.75707	.37638	.00000	.00000	5706.5	.58337	.33440	5760.5	.58890	.33449	5724.0	.58516	.33449
6	13.675	999.287	1134.76	1.13573	.75703	.00000	.00000	8971.9	.91714	.66817	9028.6	.92293	.66853	8990.3	.91901	.66835
8	13.125	999.360	1513.19	1.51438	1.13569	.00000	.00000	12247.1	1.25185	1.00289	12300.9	1.25735	1.00294	12264.5	1.25363	1.00296
10	12.766	999.405	1891.43	1.89285	1.51415	.00000	.00000	15517.9	1.58610	1.33714	15572.1	1.59165	1.33724	15533.2	1.58767	1.33700
12	12.527	999.434	2269.78	2.27144	1.89274	.00000	.00000	18797.9	1.92131	1.67234	18853.2	1.92695	1.67254	18815.6	1.92311	1.67244
17	12.089	999.486	3215.55	3.21775	2.83906	.00000	.00000	26962.0	2.75559	2.50662	26999.5	2.75942	2.50501	26979.8	2.75741	2.50674

RUN 2

								PROBE 1			PROBE 2			PROBE 3		
1	13.466	999.315	189.12	.18928	.00000	.24600	.00000	788.9	.08065	.00000	780.6	.07979	.00000	889.9	.09097	.00000
2	12.532	999.434	567.56	.56797	.37870	.58200	.33600	4073.8	.41638	.33573	4061.3	.41510	.33531	4171.0	.42631	.33534
3	12.162	999.477	945.85	.94650	.75723	.91700	.67100	7337.5	.74993	.66928	7324.4	.74858	.66879	7436.0	.75999	.66902
4	12.119	999.482	1324.44	1.32535	1.13607	1.25200	1.00600	10613.5	1.08474	1.00409	10596.3	1.08298	1.00319	10718.2	1.09544	1.00447
5	12.069	999.488	1703.12	1.70428	1.51500	1.58700	1.34100	13890.9	1.41969	1.33904	13872.1	1.41777	1.33798	13995.2	1.43035	1.33938
6	11.981	999.498	2081.34	2.08274	1.89347	1.92270	1.67670	17167.6	1.75456	1.67392	17150.4	1.75281	1.67301	17261.7	1.76418	1.67321
7	11.827	999.516	2459.41	2.46103	2.27176	2.25800	2.01200	20433.2	2.08828	2.00763	20422.3	2.08716	2.00737	20540.5	2.09924	2.00827
8	11.797	999.519	2837.87	2.83974	2.65046	2.59200	2.34600	23698.3	2.42195	2.34131	23681.0	2.42019	2.34040	23811.7	2.43355	2.34258
9	11.726	999.527	3216.18	3.21828	3.02900	2.92700	2.68100	26971.2	2.75642	2.67578	26952.9	2.75455	2.67476	27083.1	2.76786	2.67689

RUN 3

1	11.946	999.502	378.30	.37855	.00000	.41520	.00000	2434.7	.24883	.00000	2421.7	.24750	.00000	2530.9	.25866	.00000
2	11.408	999.561	756.26	.75673	.37817	.74900	.33380	5705.6	.58308	.33425	5689.7	.58146	.33396	5800.2	.59276	.33409
3	11.113	999.592	1134.51	1.13519	.75664	1.08350	.66830	8970.7	.91674	.66791	8953.2	.91495	.66745	9066.1	.92649	.66782
4	10.966	999.607	1512.88	1.51377	1.13522	1.41920	1.00400	12250.9	1.25193	1.00309	12233.0	1.25010	1.00260	12348.5	1.26190	1.00324
5	10.953	999.609	1891.30	1.89241	1.51386	1.75360	1.33840	15524.1	1.58642	1.33758	15494.1	1.58335	1.33585	15616.6	1.59587	1.33721
6	11.012	999.603	2269.52	2.27086	1.89231	2.08920	1.67400	18801.2	1.92131	1.67248	18781.2	1.91927	1.67177	18909.4	1.93237	1.67371
7	11.075	999.596	2647.76	2.64934	2.27078	2.42400	2.00880	22063.2	2.25467	2.00583	22044.8	2.25279	2.00529	22168.9	2.26547	2.00681
8	11.033	999.601	3026.03	3.02782	2.64927	2.75800	2.34280	25326.8	2.58817	2.33934	25306.9	2.58613	2.33863	25433.4	2.59906	2.34040
9	11.046	999.599	3215.21	3.21712	2.83857	2.92600	2.51080	26966.9	2.75578	2.50694	26949.0	2.75394	2.50644	27076.6	2.76698	2.50832

RUN 4

1	11.308	999.572	378.25	.37848	.00000	.41370	.00000	2433.1	.24865	.00000	2418.7	.24718	.00000	2487.3	.25419	.00000
2	11.170	999.586	756.38	.75683	.37836	.74920	.33550	5706.1	.58312	.33447	5689.2	.58139	.33421	5760.9	.58872	.33454
3	10.895	999.614	1134.71	1.13537	.75689	1.08350	.66980	8972.2	.91687	.66822	8952.8	.91488	.66770	9028.3	.92260	.66842
4	10.956	999.606	1138.49	1.13916	.76068	1.08670	.67300	9006.2	.92035	.67170	8989.0	.91859	.67141	9063.8	.92624	.67205
5	10.984	999.606	1142.27	1.14294	.76447	1.09030	.67660	9040.1	.92381	.67517	9021.9	.92195	.67477	9092.5	.92916	.67498
6	11.044	999.599	1161.17	1.16186	.78339	1.10680	.69310	9202.0	.94037	.69172	9184.7	.93859	.69141	9261.6	.94645	.69226
7	11.098	999.594	1180.08	1.18078	.80231	1.12400	.71030	9367.4	.95727	.70863	9348.4	.95533	.70815	9428.2	.96349	.70930
8	11.122	999.591	1183.86	1.18457	.80609	1.12730	.71360	9401.3	.96074	.71209	9380.8	.95864	.71146	9460.1	.96675	.71256
9	11.160	999.588	1187.64	1.18836	.80988	1.13070	.71700	9434.5	.96414	.71549	9414.1	.96205	.71487	9493.6	.97017	.71599
10	11.235	999.580	1206.54	1.20728	.82880	1.14730	.73360	9593.8	.98042	.73177	9578.4	.97885	.73167	9655.4	.98672	.73253
11	11.249	999.578	1207.49	1.20823	.82975	1.14850	.73480	9602.0	.98126	.73261	9782.4	.99969	.75251	9666.3	.98783	.73364
12	11.271	999.576	1209.38	1.21012	.83164	1.14990	.73620	9618.2	.98292	.73428	9602.4	.98131	.73413	9682.8	.98952	.73534
13	11.292	999.574	1212.21	1.21296	.83448	1.15230	.73860	9643.0	.98546	.73681	9626.0	.98371	.73653	9708.0	.99210	.73791
14	11.332	999.570	1216.00	1.21675	.83827	1.15540	.74170	9676.4	.98887	.74023	9657.3	.98693	.73974	9739.7	.99534	.74116
15	11.476	999.554	1594.04	1.59504	1.21656	1.49050	1.07680	12946.6	1.32309	1.07444	12926.3	1.32101	1.07383	13004.9	1.32904	1.07486
16	11.694	999.508	1972.44	1.97376	1.59528	1.82440	1.41070	16218.3	1.65752	1.40887	16193.3	1.65496	1.40778	16268.0	1.66260	1.40841
17	11.696	999.530	2350.83	2.35236	1.97388	2.16080	1.74710	19498.1	1.99267	1.74402	19478.5	1.99066	1.74348	19561.5	1.99914	1.74496
18	11.511	999.550	2729.11	2.73084	2.35236	2.49500	2.08130	22758.3	2.32581	2.07716	22739.6	2.32389	2.07671	22823.7	2.33249	2.07831
19	11.364	999.566	3109.58	3.11151	2.73303	2.83200	2.41830	26048.3	2.66199	2.41335	26031.1	2.66023	2.41305	26114.6	2.66877	2.41458

RUN 5

1	12.219	999.471	189.19	.18932	.00000	.24620	.00000	792.2	.08096	.00000	805.6	.08234	.00000	853.4	.08722	.00000
2	11.841	999.514	378.35	.37860	.18927	.41410	.16790	2432.9	.24865	.16769	2447.7	.25016	.16782	2493.8	.25487	.16765
3	11.696	999.530	567.44	.56781	.37849	.58230	.33610	4074.5	.41641	.33545	4086.8	.41767	.33534	4134.1	.42250	.33528
4	11.374	999.565	756.64	.75711	.56779	.74950	.50330	5707.6	.58329	.50233	5717.6	.58432	.50198	5753.5	.58799	.50076
5	11.102	999.594	945.90	.94646	.75714	.91650	.67030	7338.5	.74995	.66898	7348.9	.75101	.66867	7391.2	.75533	.66811
6	10.909	999.613	1135.12	1.13578	.94646	1.08400	.83780	8975.0	.91716	.83620	8977.3	.91740	.83506	9029.7	.92276	.83554
7	10.793	999.625	1324.24	1.32500	1.13567	1.25140	1.00520	10611.6	1.08440	1.00343	10619.4	1.08519	1.00286	10668.1	1.09017	1.00295
8	10.691	999.635	1513.33	1.51418	1.32486	1.41900	1.17280	12248.5	1.25166	1.17070	12257.0	1.25253	1.17020	12307.8	1.25773	1.17050
9	10.621	999.641	1702.57	1.70352	1.51420	1.58650	1.34030	13886.7	1.41906	1.33809	13890.3	1.41943	1.33709	13941.7	1.42468	1.33746
10	10.691	999.635	1891.82	1.89290	1.70357	1.75400	1.50780	15522.7	1.58625	1.50528	15524.0	1.58638	1.50405	15574.9	1.59158	1.50436
11	10.808	999.623	2080.97	2.08217	1.89285	1.92250	1.67630	17163.9	1.75397	1.67301	17166.1	1.75420	1.67187	17224.5	1.76017	1.67295
12	10.746	999.629	2270.13	2.27143	2.08211	2.08990	1.84370	18800.7	1.92123	1.84027	18805.2	1.92169	1.83936	18862.7	1.92756	1.84034
13	10.665	999.637	2459.20	2.46059	2.27127	2.25720	2.01100	20430.8	2.08779	2.00682	20436.5	2.08838	2.00604	20499.0	2.09476	2.00753
14	10.612	999.642	2648.29	2.64977	2.46045	2.42430	2.17810	22061.2	2.25439	2.17342	22067.8	2.25506	2.17273	22128.9	2.26130	2.17408
15	10.588	999.645	2837.47	2.83905	2.64973	2.59170	2.34550	23694.7	2.42130	2.34033	23701.6	2.42200	2.33967	23762.6	2.42824	2.34102
16	10.567	999.647	3026.60	3.02829	2.83896	2.75850	2.51230	25326.5	2.58804	2.50708	25332.9	2.58869	2.50636	25395.5	2.59509	2.50787
17	10.542	999.649	3215.86	3.21764	3.02832	2.92640	2.68020	26966.9	2.75566	2.67469	26974.0	2.75638	2.67405	27037.1	2.76284	2.67561

KEY: Probe 1 XR 38
Probe 2 DDR 6000
Probe 3 B&H Electromanometer

UNITS: Temperature degrees Celsius
Density kg/m^3
Volume $(meters)^3$
Height meters
Pressure pascals
Mass kg

The abbreviation TR stands for volume transfer

* No sight glass observations made during Run 1
** Temperature corrections applied to sight glass for Runs 6, 7, and 8 (see text).

Table 2: Data shown are collected from each of the eight calibration runs on which the reported analysis is based.
Table continued on next page.

RUN 6

TR NO.	TANK TEMP.	WATER DENSITY	WATER MASS	ACCUM. VOLUME	DELTA VOLUME	SIGHT GLASS HEIGHT	DELTA HEIGHT	PROBE 1 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT	PROBE 2 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT	PROBE 3 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT
1	10.156	999.685	207.95	.20806	.00000	.26450**	.00000	959.6	.09806	.00000	969.9	.09910	.00000	1011.5	.10335	.00000
2	10.284	999.673	397.19	.39741	.18935	.42982	.16532	2597.5	.26542	.16737	2610.0	.26669	.16759	2652.4	.27103	.16768
3	10.549	999.648	586.34	.58667	.37861	.59900	.33450	4235.9	.43284	.33479	4249.6	.43425	.33515	4292.5	.43863	.33528
4	10.745	999.629	775.40	.77584	.56779	.76556	.50106	5866.0	.59943	.50138	5880.4	.60090	.50180	5923.6	.60532	.50197
5	10.746	999.629	964.49	.96504	.75698	.93232	.66782	7498.4	.76624	.66818	7507.8	.76720	.66810	7554.1	.77193	.66857
6	10.732	999.631	1153.62	1.15428	.94622	.99955	.73505	9138.0	.93378	.83573	9145.4	.93455	.83544	9190.2	.93912	.83576
7	10.680	999.636	1342.73	1.34349	1.13543	1.26644	1.00194	10775.1	1.10106	1.00301	10784.6	1.10204	1.00294	10825.5	1.10621	1.00286
8	10.657	999.638	1531.82	1.53269	1.32463	1.43331	1.16881	12409.7	1.26810	1.17004	12419.8	1.26913	1.17003	12460.6	1.27330	1.16994
9	10.645	999.639	1721.02	1.72199	1.51393	1.60051	1.33601	14043.7	1.43507	1.33702	14054.5	1.43618	1.33707	14096.8	1.44050	1.33715
10	10.631	999.640	1910.25	1.91132	1.70326	1.76734	1.50284	15677.7	1.60204	1.50398	15685.3	1.60282	1.50372	15733.1	1.60770	1.50435
11	10.560	999.647	2099.38	2.10055	1.89250	1.93522	1.67072	17320.2	1.76986	1.67180	17328.9	1.77075	1.67165	17373.4	1.77530	1.67195
12	10.607	999.643	2288.60	2.28988	2.08183	2.10284	1.83834	18960.7	1.93750	1.83945	18966.6	1.93811	1.83901	19018.2	1.94338	1.84003
13	10.658	999.638	2477.75	2.47914	2.27109	2.26971	2.00521	20594.8	2.10450	2.00644	20597.9	2.10481	2.00571	20650.1	2.11015	2.00680
14	10.656	999.638	2666.90	2.66840	2.46035	2.43597	2.17147	22224.2	2.27100	2.17294	22231.1	2.27170	2.17260	22283.2	2.27702	2.17367
15	10.605	999.643	2856.13	2.85773	2.64968	2.60310	2.33860	23853.1	2.43744	2.33938	23862.9	2.43844	2.33934	23916.2	2.44388	2.34053
16	10.563	999.647	3045.33	3.04703	2.83897	2.77050	2.50600	25483.2	2.60399	2.50594	25493.7	2.60507	2.50597	25548.0	2.61062	2.50727
17	10.519	999.651	3234.54	3.23633	3.02827	2.93719	2.67269	27125.0	2.77176	2.67370	27134.8	2.77275	2.67365	27190.5	2.77845	2.67509

RUN 7

TR NO.	TANK TEMP.	WATER DENSITY	WATER MASS	ACCUM. VOLUME	DELTA VOLUME	SIGHT GLASS HEIGHT	DELTA HEIGHT	PROBE 1 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT	PROBE 2 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT	PROBE 3 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT
1	12.056	999.490	245.73	.24590	.00000	.29720	.00000	1286.9	.13153	.00000	1296.9	.13254	.00000	1339.3	.13688	.00000
2	11.920	999.505	434.82	.43511	.18922	.46453	.16733	2921.3	.29856	.16703	2933.1	.29976	.16722	2974.6	.30400	.16712
3	11.853	999.513	624.06	.62447	.37858	.63272	.33552	4560.8	.46611	.33459	4576.6	.46773	.33519	4618.1	.47197	.33509
4	11.788	999.520	813.24	.81378	.56788	.79870	.50150	6191.7	.63278	.50126	6204.5	.63409	.50155	6248.7	.63861	.50173
5	11.632	999.537	1002.47	1.00311	.75722	.96581	.66861	7826.7	.79987	.66834	7836.8	.80089	.66835	7881.1	.80542	.66854
6	11.516	999.550	1191.41	1.19216	.94627	1.13284	.83564	9465.5	.96733	.83580	9471.5	.96795	.83540	9516.8	.97258	.83570
7	11.409	999.561	1380.60	1.38146	1.13556	1.29964	1.00244	11103.4	1.13471	1.00318	11107.2	1.13510	1.00255	11155.0	1.13998	1.00310
8	11.305	999.572	1569.69	1.57066	1.32476	1.46670	1.16950	12736.6	1.30159	1.17007	12744.4	1.30239	1.16985	12791.0	1.30715	1.17027
9	11.214	999.582	1758.89	1.75996	1.51406	1.63392	1.33672	14371.1	1.46861	1.33709	14378.1	1.46934	1.33679	14424.9	1.47411	1.33723
10	11.129	999.591	1948.03	1.94920	1.70331	1.80183	1.50463	16004.0	1.63547	1.50395	16009.4	1.63603	1.50348	16058.6	1.64105	1.50417
11	11.044	999.599	2137.30	2.13857	1.89267	1.96869	1.67149	17648.1	1.80347	1.67194	17655.9	1.80427	1.67172	17703.2	1.80909	1.67222
12	10.977	999.606	2326.53	2.32790	2.08200	2.13612	1.83892	19289.4	1.97119	1.83966	19294.6	1.97171	1.83917	19343.6	1.97672	1.83984
13	10.910	999.613	2515.72	2.51719	2.27129	2.30290	2.00570	20920.4	2.13784	2.00631	20926.4	2.13845	2.00590	20976.1	2.14353	2.00665
14	10.853	999.619	2704.90	2.70647	2.46058	2.46930	2.17210	22551.6	2.30452	2.17299	22558.2	2.30519	2.17264	22607.0	2.31018	2.17330
15	10.799	999.624	2894.20	2.89587	2.64997	2.63717	2.33997	24181.1	2.47102	2.33950	24191.9	2.47212	2.33958	24239.4	2.47698	2.34010
16	10.748	999.629	3083.32	3.08508	2.83919	2.80374	2.50654	25818.3	2.63830	2.50678	25829.6	2.63946	2.50691	25880.3	2.64465	2.50777

RUN 8

TR NO.	TANK TEMP.	WATER DENSITY	WATER MASS	ACCUM. VOLUME	DELTA VOLUME	SIGHT GLASS HEIGHT	DELTA HEIGHT	PROBE 1 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT	PROBE 2 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT	PROBE 3 PRESSURE	ACCUM. HEIGHT	DELTA HEIGHT
1	14.518	999.169	113.44	.11355	.00000	.17870	.00000	128.0	.01308	.00000	136.8	.01399	.00000	190.8	.01951	.00000
2	12.152	999.479	302.58	.30279	.18924	.34711	.16841	1777.7	.18169	.16860	1787.2	.18266	.16868	1837.8	.18783	.16832
3	12.006	999.495	491.68	.49201	.37846	.51421	.33551	3413.4	.34886	.33578	3421.9	.34973	.33575	3469.6	.35460	.33510
4	12.002	999.496	680.87	.68133	.56779	.68218	.50348	5055.6	.51669	.50361	5063.5	.51751	.50352	5111.6	.52242	.50291
5	11.867	999.511	869.98	.87056	.75701	.84865	.66995	6685.2	.68324	.67015	6694.3	.68417	.67018	6739.5	.68878	.66927
6	11.696	999.530	1059.26	1.05995	.94640	1.01564	.83694	8319.0	.85019	.83711	8325.1	.85082	.83684	8375.2	.85594	.83643
7	11.597	999.541	1248.40	1.24920	1.13565	1.18305	1.00435	9956.2	1.01750	1.00442	9962.8	1.01818	1.00420	10013.8	1.02339	1.00388
8	11.519	999.550	1437.47	1.43838	1.32483	1.35029	1.17159	11593.0	1.18478	1.17169	11599.5	1.18544	1.17145	11650.6	1.19066	1.17115
9	11.454	999.557	1626.64	1.62766	1.51412	1.51715	1.33845	13231.1	1.35217	1.33908	13235.2	1.35259	1.33861	13286.4	1.35782	1.33831
10	11.395	999.563	1815.80	1.81693	1.70339	1.68392	1.50522	14867.1	1.51936	1.50627	14870.9	1.51975	1.50576	14919.4	1.52470	1.50519
11	11.341	999.569	2004.88	2.00612	1.89257	1.85131	1.67261	16507.2	1.68696	1.67387	16510.5	1.68730	1.67331	16559.7	1.69232	1.67281
12	11.283	999.575	2194.07	2.19542	2.08187	2.01858	1.83988	18142.5	1.85407	1.84098	18147.7	1.85460	1.84062	18198.2	1.85976	1.84025
13	11.243	999.579	2383.24	2.38470	2.27115	.00000	.00000	19776.8	2.02107	2.00799	19782.9	2.02170	2.00772	19835.0	2.02702	2.00752
14	11.202	999.583	2572.43	2.57399	2.46044	.00000	.00000	21406.8	2.18764	2.17456	21413.7	2.18835	2.17437	21467.0	2.19379	2.17429
15	11.174	999.586	2761.60	2.76327	2.64972	.00000	.00000	23039.9	2.35453	2.34145	23045.0	2.35505	2.34107	23099.5	2.36062	2.34111
16	11.135	999.590	2950.78	2.95256	2.83901	2.68590	2.50720	24671.6	2.52127	2.50819	24679.3	2.52205	2.50807	24731.3	2.52737	2.50786
17	11.095	999.594	3139.96	3.14184	3.02829	2.85301	2.67431	26311.4	2.68884	2.67575	26318.9	2.68960	2.67562	26372.4	2.69507	2.67557

Table 2 (Continued)

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8
Slopes (cm/liter)								
Sight Glass	.088469 (35)	.088492 (18)	.088484 (17)	.088464 (13)	.088493 (12)	.088294** (16)	.088282** (12)	.088261* (12)
XR-38	.088300 (28)	.088311 (18)	.088312 (25)	.088310 (11)	.088303 (13)	.088292 (15)	.088309 (15)	.088311 (15)
Ruska DDR 6000	.088235 (46)	.088258 (59)	.088303 (27)	.088304 (11)	.088280 (13)	.088278 (12)	.088292 (13)	.088301 (11)
B & H Electromanometer	.088297 (28)	.088368 (15)	.088380 (33)	.088340 (18)	.088361 (17)	.088326 (12)	.088325 (13)	.088318 (11)
Intercepts (cm)								
Sight Glass	-.008 (57)	.085 (35)	-.081 (32)	.040 (16)	.059 (22)	-.047 (25)	.035 (21)	.187 (21)
XR-38	.014 (46)	.106 (34)	.037 (48)	.004 (13)	.085 (23)	.047 (28)	.008 (26)	.177 (25)
Ruska DDR 6000	.086 (22)	.175 (112)	-.012 (50)	-.030 (14)	.055 (23)	.061 (22)	.019 (22)	.167 (20)
B & H Electromanometer	.034 (45)	.039 (28)	-.024 (63)	.031 (23)	-.018 (32)	.042 (23)	.014 (22)	.102 (19)
Residuals S.D.*								
Sight Glass	.069	.045	.041	.035	.042	.048	.031	.038
XR-38	.055	.043	.061	.028	.044	.054	.048	.047
Ruska DDR 6000	.091	.144	.063	.030	.045	.042	.041	.036
B & H Electromanometer	.054	.036	.079	.048	.061	.043	.040	.035
Standard Deviation of a Predicted Point (Maximum)								
Sight Glass	.057	.029	.027	.023	.020	.027	.019	.019
XR-38	.045	.028	.040	.019	.021	.026	.024	.028
Ruska DDR 6000	.075	.093	.041	.020	.021	.020	.020	.022
B & H Electromanometer	.045	.024	.052	.032	.029	.021	.020	.021
Number of Calibration Points	7	9	9	19	17	17	16	17

* Includes both tank configuration and measurement error

** Sight glass levels corrected for temperature

Table 3: Summary of slopes, intercepts and residual standard deviations for all eight runs. (Figures in parentheses are standard errors of values immediately above.)

a: Intercept of the calibration line;

b: Slope of the calibration line;

E: Deviations from calibration line due to tank configuration;

e: Deviation due to measurement error.

The results are summarized in table 3 for the eight runs. Run 1, however, cannot be considered as a regular test since the XR - 38 was used for all probes. The residual standard deviation thus obtained include contributions from both E, the tank configuration errors, and e, the measurement error associated with the instrument. The decomposition and estimation of the error term will be discussed later. It will suffice to note here that the tank configuration error should remain constant at a given level of liquid whereas measurement errors may be assumed to be random.

The values of the slope of the calibration lines obtained from the sight glass during the first five tests are higher than the other three instruments by about .0002, indicating the presence of a systematic effect. This was traced to the difference in temperatures, and hence density, of the water inside the tank and in the sight glass (see next section and appendix 1 for detailed analysis). Corrections of this temperature difference were possible for runs 6, 7, and 8 where the temperature of the water in the sight glass was measured. The slopes of these three runs agree well with the other three instruments after making appropriate temperature corrections.

We observe from table 3 the following:

1. For runs 2 through 8, the Bell and Howell Electromanometer gave consistently higher slopes than the XR-38, the DDR 6000, and the sight glass after temperature correction.

2. The intercepts for run 8 are higher than those of the other runs.

3. The residual standard deviations are of about the same magnitude. The large value for the Ruska DDR 6000 for run 2 may be traced to a possible transposition of a reading in the last two digits. This "standard deviation" however includes a component due to the deviation of the tank from the model geometry.

In fitting the response of an individual instrument's calibration lines for a run, the plot of each set of residuals suggested that there is a definite "profile" of the tank which is common to all four instruments. It would be desirable to plot such a profile using more than one run to sketch the tank configuration in detail. Runs 5, 6, 7 and 8 were chosen for this purpose.

To align the four runs to the same starting point, the initial volume increments, varying from 113 liters for run 8 to 245 liters for run 7, were multiplied by the respective calibration line slopes obtained for each instrument. This resulted in liquid levels which could be added to the accumulated heights for each instrument in a run such that all four runs were referred to a common starting point. A total of sixty-two points was used from the four runs Table 4 gives the volume in liters in column 1, residuals from the calibration line for each instrument in columns 2, 3, 4, and 5, and the averages of residuals in column 6. Figure 2 is a plot of all the residuals from the four instruments against tank volume in liters, and figure 3 is a plot of the averages of the residuals against tank volume.

From these two figures, it is evident that the spread of the points at a given volume is a measure of the measurement errors of the instruments; whereas the deviations of the averages of these points from the fitted line is a measure of constancy of the tank diameter from bottom to top, i.e., a profile of the tank. The two peaks correspond to the lower two of the three braces supporting the probes, the third one is at the top of the tank and is outside the range shown in the figure. The systematic error in using the calibration lines for the estimation of volume from instrument readings is less than 1 liter.

Since each deviation (E) of the tank configuration is systematic in nature, the measurement error standard deviation, σ_e, can be estimated from the residual standard deviations. Denoting by δ the differences between the residuals of an instrument from the average of all four instruments, e.g., (column 1 - column 6 of table 4), then:

$$\delta = e_1 - \frac{e_1 + e_2 + e_3 + e_4}{4}$$

VOLUME	SIGHT GLASS	XR 38	DDR 6000	B & H	AVERAGED RESIDUALS
302.77200	-.019762537	-.130451232	-.0084827087	.015226619	-.010867465
378.56600	-.018942847	-.028432959	.015535164	.063549553	.0079272275
397.36200	-.055270519	-.025343254	-.016881560	.0035515352	-.023485949
435.07300	-.0068540543	-.017524946	-.0064380068	-.016355022	-.011793007
492.00200	-.024243108	-.021080597	-.017365375	-.017375452	-.020016133
567.77599	.057305119	.033817504	.062004662	.10479072	.064479502
586.48199	.062326396	.016870362	.038035875	.059416573	.044162301
624.43299	.088637871	.020264689	.074532076	.058505278	.060484978
681.31200	.054201870	.041226892	.046690009	.042957505	.046269069
757.07599	.025590873	.0081233357	.020531537	-.071919041	-.0044183239
775.66199	.0096138337	-.026213204	.0076569868	.019982187	.0027599508
813.73299	-.030563300	-.036649811	-.0092035852	-.0013375451	-.019438310
870.54199	-.010277602	-.0094013753	.0078091267	-.029644918	-.010378692
946.43099	-.030993490	-.042428575	-.025797647	-.073490894	-.043177652
964.85199	-.013981535	-.050180146	-.063606507	-.030336598	-.039526196
1003.0730	-.033300876	-.047091796	-.036469738	-.044713065	-.040393868
1059.9420	-.039792074	-.045043671	-.056083223	-.047262228	-.047045299
1135.7490	-.034301820	-.039713401	-.098860496	-.051792290	-.056167001
1154.0920	-.011999379	-.0085608177	-.039280423	-.025071020	-.021227910
1192.1230	-.020396147	.0080726177	-.028130235	-.022472885	-.015731662
1249.1820	.0048440325	-.026554105	-.025848087	-.010747914	-.014576518
1324.9640	-.038496143	-.027903590	-.022829707	-.030993283	-.030055681
1343.3020	-.017365274	.015705534	.0076912631	-.027155276	-.0052809382
1381.4130	-.048713202	.032041829	-.020983124	-.0014328423	-.0097718351
1438.3620	.024785969	-.0027674941	-.00031478079	.011067429	.0081927805
1514.1530	-.020390740	-.0037973195	.0054936823	.0021017893	-.0041481470
1532.5020	-.021844435	.010856365	.015545150	-.038358073	-.0084502483
1570.6130	-.039071884	.013958890	.0041136051	.0075571298	-.0033605650
1627.6520	.0049984066	.021304400	-.0044958628	.0031643123	.0062428141
1703.4920	-.025557410	.017063968	-.019421897	-.028054409	-.013992437
1721.8020	-.015169855	-.0028238579	.014571321	-.038392917	-.010453827
1759.9130	-.028273775	-.0029535744	-.019622470	-.022284886	-.018283676
1816.9220	-.023020522	.027142178	.0030913477	-.022972712	-.0039399271
1892.8650	-.033733995	.014921354	-.047340162	-.071218166	-.034342742
1911.1320	-.041149575	-.019151089	-.039050233	-.041077683	-.035107145
1949.1630	.056945958	-.035450456	-.058941895	-.037711766	-.018789540
2006.1020	.016922154	.080929757	.048625392	.028841691	.043829748
2082.1370	.067029017	.071699879	.033657789	.054545704	.056733097
2100.3720	.040832505	.052467017	.055273638	.0041888095	.038190492
2138.5230	.032439473	.042338708	.042027259	.037149055	.038488624
2195.4020	.046253207	.074119694	.063563590	.060058115	.060998651
2271.3970	.058851387	.089535177	.075714152	.071367349	.073867016
2289.7020	.094854432	.10614120	.071655407	.091507825	.091039716
2327.8530	.070584951	.10277638	.075643276	.074656484	.080915273
2460.5560	.049612025	.036289827	.036687389	.077115600	.049926210
2478.9620	.075067237	.085993398	.034212738	.055004149	.062569380
2517.1430	.052268566	.046748014	.032793493	.045697727	.044376950
2573.9720	.027566318	.0031510639	.016088838	.025139950	.017986542
2649.7420	.017983956	-.0093403099	.0052764385	.020475887	.0085989928
2668.2220	-.0047213308	.025844679	.016768619	.028502609	.016598644
2706.4230	-.025162718	.0016013598	-.0091749721	-.012376262	-.011278148
2763.2520	-.021335028	-.021892545	-.027207611	-.011878108	-.020578323
2839.0210	.0081263121	-.033180421	-.0043424116	-.014381117	-.010944409
2857.5520	-.010699156	-.040482262	-.026851272	-.014179730	-.023053105
2895.8230	-.063206853	-.074141806	-.031736790	-.061051519	-.057534242
2952.5420	-.031120490	-.067820380	-.041387050	-.049780038	-.047526990
3028.2530	-.057569844	-.062872565	-.039810986	-.045085283	-.051334669
3046.8420	-.073138111	-.10327904	-.066941567	-.053330561	-.074172318
3085.0430	-.0053341471	-.053991228	-.0084090304	-.0038300182	-.017891106
3141.8220	-.010025177	-.022867328	-.0046885158	.0032009305	-.0085950226
3217.6120	-.024511465	-.023780970	.0035034254	-.0070111942	-.012950051
3236.1520	-.017352360	-.037842021	-.0087965659	.0057510412	-.014559976

Table 4: Above are the volume increments and residuals from the fitted calibration line for the combined runs 5 - 8. Data has been adjusted to have the same zero. The volume is given in liters and the residuals in centimeters.

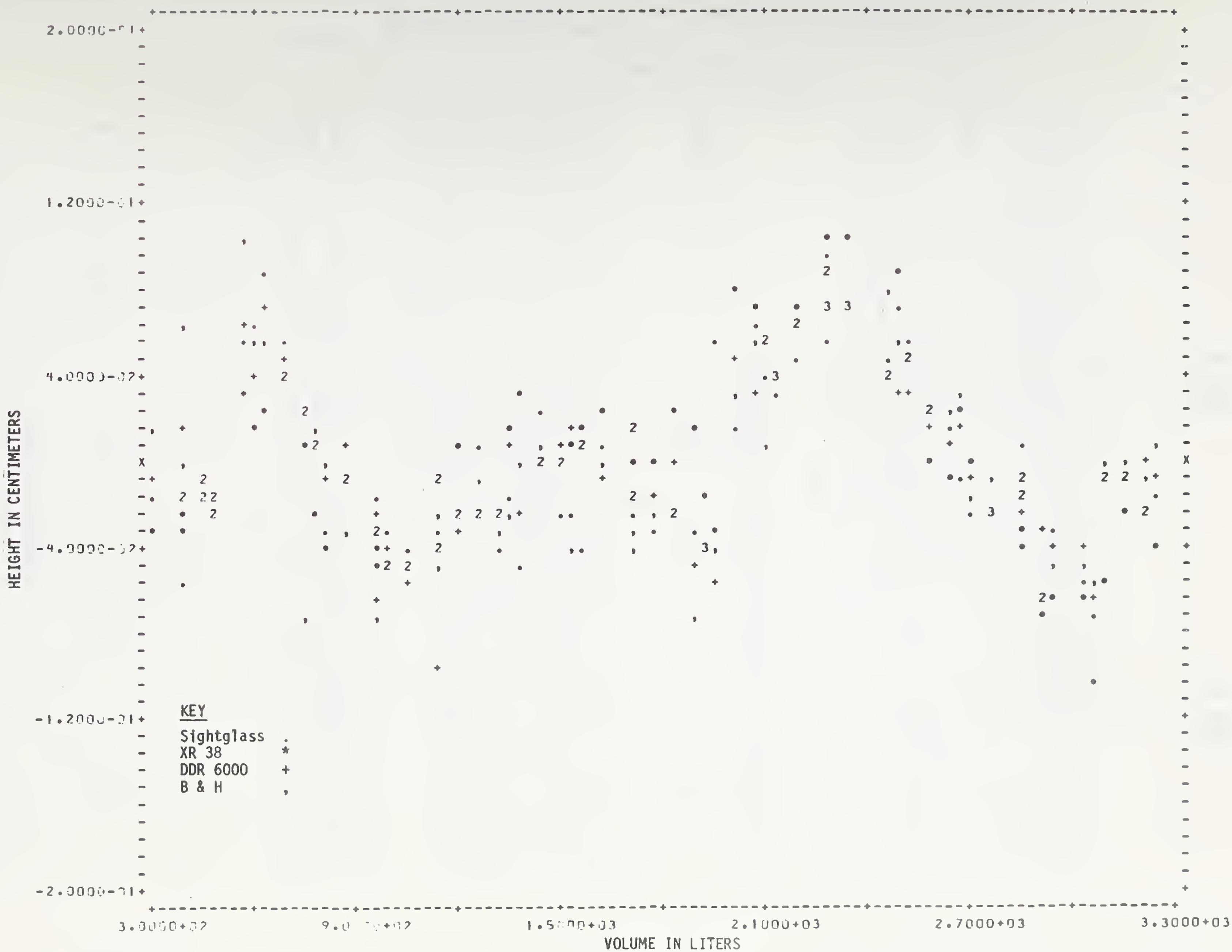


Figure 2: Plotted above are the residuals from the straight line fit to the calibrated responses from the four level detectors for runs 5 - 8.

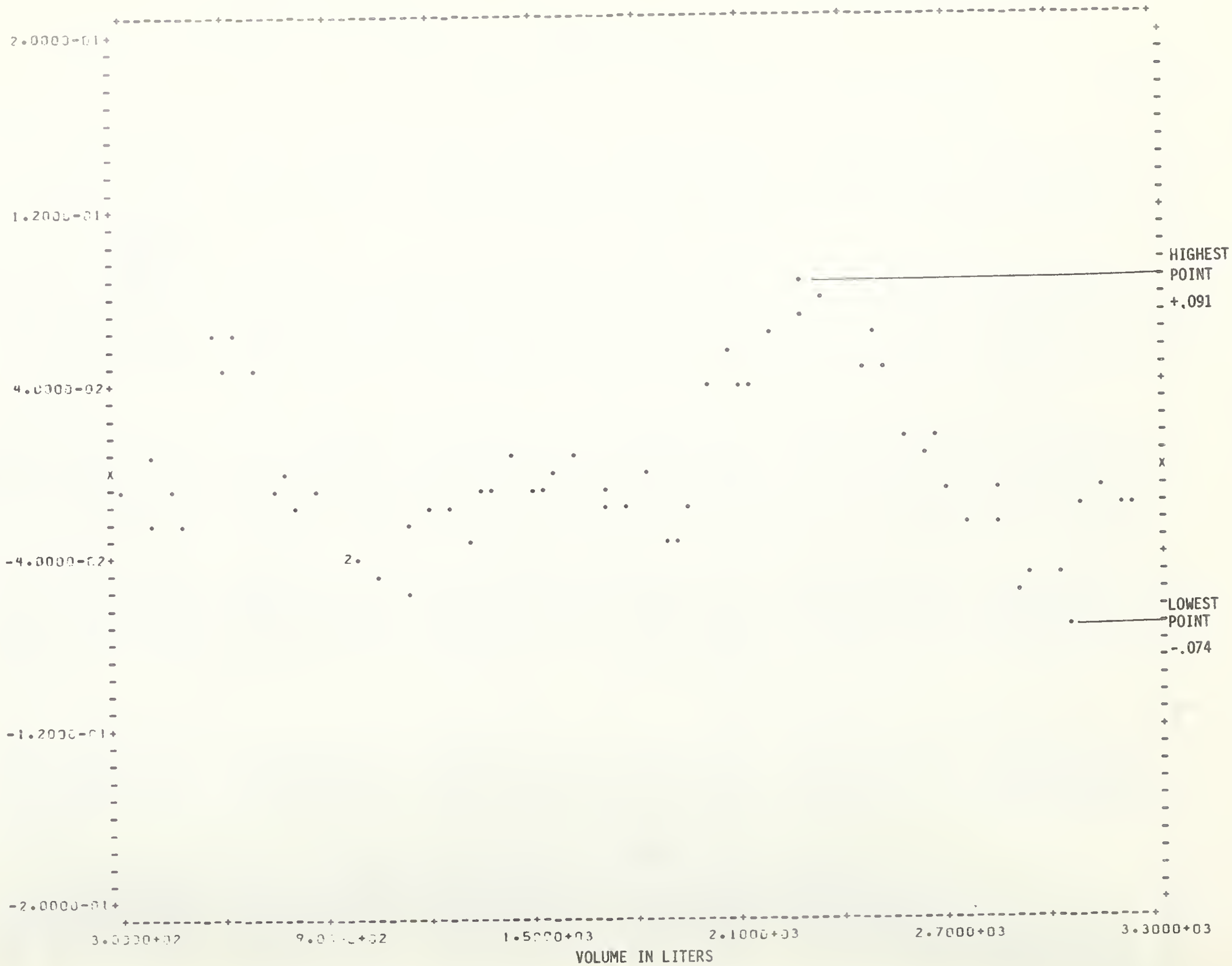


Figure 3: Plotted above are the averages for the residuals shown in figure 2. The curve is descriptive of the tank's departures from right circular geometry.

where the subscripts refer to the instruments. The tank configuration error, E, cancels out since it is the same for both individual instrument readings and the average of four readings. Hence, the variance of δ:

$$\mathrm{Var}(\delta) = 9/16\ \mathrm{Var}(e_1) + (1/16\ \mathrm{Var}(e_2) + 1/16\ \mathrm{Var}(e_3) + 1/16\ \mathrm{Var}\ e_4)$$

Table 5 lists the residual standard deviation, the standard deviation corresponding to measurement errors.

Table 5

	Sight Glass (cm)	XR-38 (cm)	DDR 6000 (cm)	Bell & Howell (cm)
Residual Std. Dev.	.0405	.0448	.0386	.0431
$\hat{\sigma}_e$	.0208	.0203	.0181	.0218

The pooled value of $\hat{\sigma}_e$ is 0.0204 centimeters. Judging from the small spread of the four values of estimated $\hat{\sigma}_e$, the assumption of equal precision of the four instruments seems to be justified. Therefore, for a 17-point calibration of this tank, the uncertainty of a point on the calibration line can be expected to be less than the sum of the bounds to systematic error due to tank configuration and the maximum width of the 95 percent confidence band about the calibration line, or equal to $0.091 + 0.027 = 0.118 \approx 0.12$, equivalently 1.4 liters.

In using the calibration line corresponding to an instrument reading, an additional error of 3 standard deviations of measurement error, or equivalently 0.7 liter, should be added giving a total uncertainty of 2 liters for a tank volume of about 3000 liters.

Summary and Conclusions

The results of this study demonstrate clearly that the uncertainty of the calibration of the volume-height relationship of a tank depends both on the calibration techniques and on the actual tank geometry. The basis for description of a tank should be a simple geometrical one which adequately accounts for the tank's gross characteristics, e.g., straight-walled vessels of appropriate cross section. With the precision available in calibration techniques and liquid level measuring instruments, the actual contours of a tank may be represented as the departure of actual tank geometry from that of the model. The conditions necessary for detection of such departures are the following:

1. The uncertainty in the volumetric increments used in the calibration should be considerably smaller than departure of the actual tank geometry from that of the model.

2. The level detection apparatus should have sufficient precision to detect departure from the model geometry.

3. Temperature measurement should be of sufficient accuracy and precision to contribute zero variability to the calibration process.

Under these circumstances, two, at the most three, properly designed calibration passes should suffice to generate sufficient information to characterize actual tank deviations from that of the geometric model. This study has met these criteria in that the uncertainty associated with volumetric increment addition and temperature measurement are an order of magnitude below that of the level detection instruments.

Based on the knowledge of the actual geometry, relative to a particular set of operating limits, it may be expedient to accept a linear calibration curve, that is, to assume that the tank is sufficiently well characterized by the geometric model. The consequence of such a decision is to take the maximum deviation of the actual tank geometry from the model geometry as the systematic error associated with the operation of the tank. However, should the limits of variability for tank operation be reduced, the information necessary to construct a more descriptive tank calibration curve is available with little additional effort. Such a calibration is limited only by the random component of variability, assuming that the systematic component has been minimized. The use of the corrections for departure of actual from model geometry in this case reduces the measurement uncertainty of the tank to 1.4 liters.

It would be remiss not to point out a mechanism for circumvention of the intent of safeguards and accountability practices. The knowledge of the magnitude and sign of actual tank deviations from the model allows the possibility for the measured volumes to be systematically incorrect. Although the uncertainty associated with a particular tank load may be within accountability limits, the accumulated uncertainty associated with the total amount of fluid processed in this manner could quickly become unacceptably large, exceeding accountability limits. Rectification of this problem is two-fold: either construct tanks whose geometry closely fits a simple geometric model or use the actual tank profile as measured by the calibration process. This calibration concept is most critical when the tank in question is the accountability tank or tanks for nuclear fuel reprocessing plants.

This exercise has clearly demonstrated the physical reality of the need for temperature correction when a manometer is operated over a region of thermal gradients. In this case, water was the manometer fluid, a fluid having a relatively small thermal expansion coefficient. In the case of manometer oils, the thermal expansion coefficients are at least twice as large and may be three to four times that of water for hydrocarbon fluids of density 0.7 to 0.8 g/cm^3. Not only does this effect influence calibration operations but also normal operation of the tank. One would expect additional complication caused by changing seasons at a plant site which could induce sizeable changes in the temperature gradient between sections of the manometer. Sight glass operation which disregards the existence of temperature gradients may contribute significant error to calibration and normal operation. Problems of this nature are not encountered with electromanometers.

Although water has been used as the calibration fluid in this work, any fluid of known density and thermal expansion properties could be used. Given the commercial availability of apparatus to measure these parameters rapidly and with sufficient accuracy, the use of fluids more suitable to a particular process would seem to pose no great difficulty.

Acknowledgments

The authors wish to express their gratitude to Leon Carroll for his help with computer programming and data table preparation and to Mrs. Gertrude Tesler for her help in the preparation of this manuscript. The interest and assistance of S.C. Suda, Brookhaven Technical Support Organization, and P.E. Pontius, Chief of the Mass and Volume Section are greatly appreciated.

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Appendix 1

Sight Glass Analysis

In practice, the sight glass is used in conjunction with the length unit by which the enclosed liquid level of a vessel can be determined. The liquid surface is visible through the tube wall and the level is compared with the attached length scale. The visible liquid surface approximates that of the enclosed surface. When the liquid level is changed, the difference in level is the difference between two scale readings.

Usually there is a reference level assignment that may coincide with an empty tank and an arbitrary head volume. Liquid level is usually expressed as the height above the reference level.

We assume the length scale is correct and without significant angular displacement from a vertical attitude. Also, capillary depression and meniscus uniformity are nearly constant over the sight glass range.

The sight glass can be considered as a manometer with a different fluid density in each leg due to nonisothermal conditions in the legs (see fig. 4). At equilibrium, the pressure of each leg must be the same. Therefore, we can write the following equation:

$$P_2 = P_1$$

where P_2 is the pressure in the sight glass leg and

P_1 is the pressure in the vessel at level H_1

These pressures can be expressed in terms of liquid density, height, and local acceleration of gravity as follows:

$$\rho_2 g H_2 = \rho_1 g H_1$$

where ρ_2 = liquid density in sight glass

ρ_1 = liquid density in vessel

H_2 = meniscus height in sight glass

H_1 = liquid height in vessel

g = local acceleration of gravity

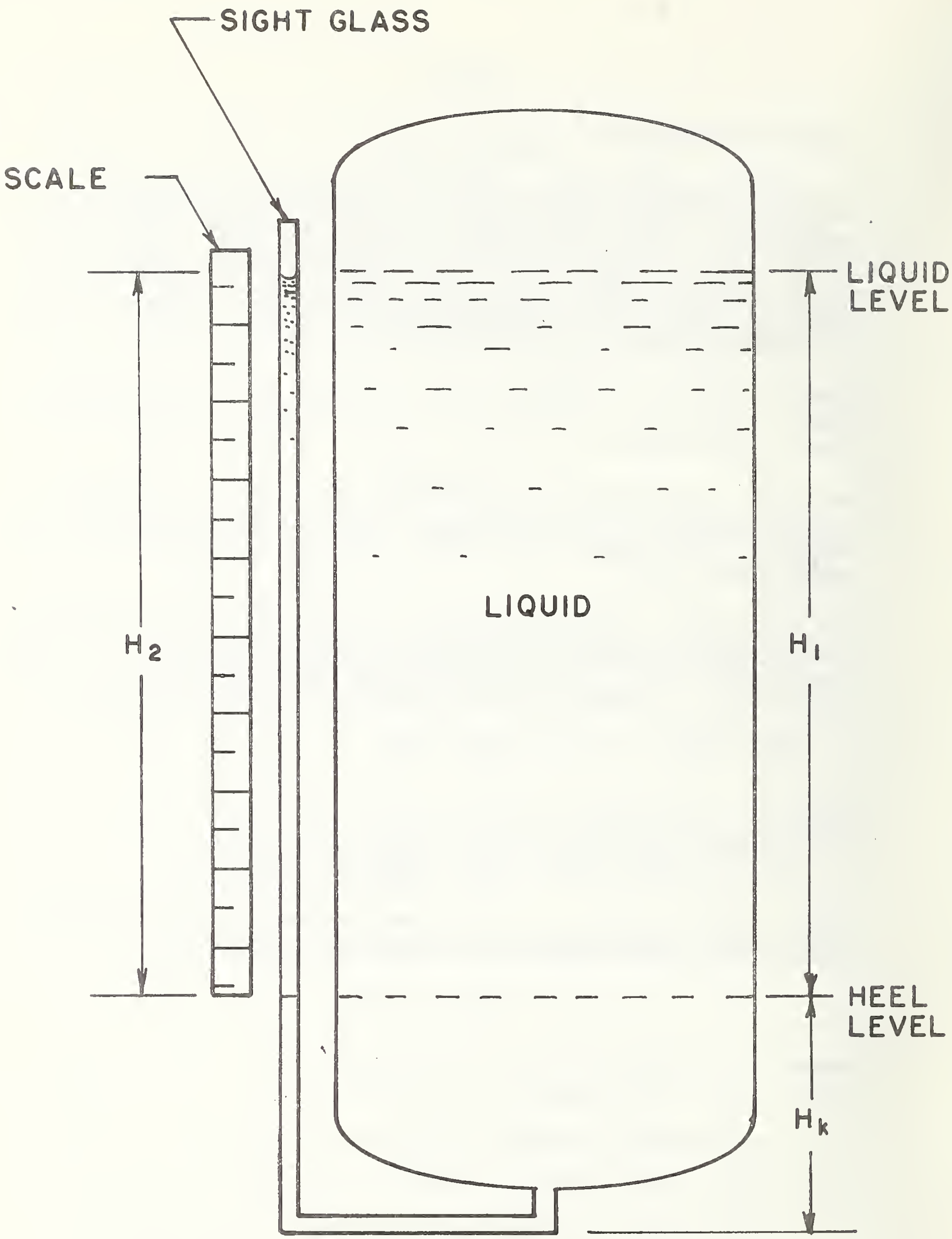

FIGURE 4: SIGHT GLASS TUBE AND SCALE ARRANGEMENT

Solving for liquid level of interest, H_1, of the vessel in terms of the observed sight glass height, H_2, we have

$$H_1 = \frac{\rho_2 H_2}{\rho_1}$$

Obviously the user of the sight glass method for determining liquid level in a vessel must know the density ratio ρ_2/ρ_1 or force the term to unity ($P_1 = P_2$) by an appropriate technique as discussed below.

If there is a heel volume to the vessel, then there is a corresponding height, H_k, that is, a constant level below which the contents are not observable and the manometer effect is still present. Then H_k must be estimated by some means and added as a constant to observations of H_1 and H_2 for the adjustment of temperature effects and hence density differences.

Sight Glass Observation

Sight glass observations [4] of liquid level are made by estimating the intersection of a horizontal plane and the length scale. The plane is tangent to the bottom of the meniscus formed at the air-liquid interface inside the tube.

Sight Glass Experiment

At first glance the sight glass appears to indicate liquid level in a tank without second order effects if properly constructed. In comparing all of the fitted slope values for the various instruments, calibrations 1 - 5 show that the sight glass slopes are systematically larger than the others. This systematic difference is about 2 parts in 10^3, a very significant error for this case.

In our experiment, as in plant operations, the processing tank may contain liquids at temperatures considerably different from the surrounding ambient conditions. When the sight glass and contained liquid are not at tank temperature, an error exists between the sight glass indicated level and the true liquid level inside the tank.

In runs 6 - 8, water temperature was measured by an immersed thermometer inside the sight tube. This experiment indicated the water in the sight glass was about 12 °C warmer than that in the tank. The sight glass data were then corrected for this error by the method based on water temperature measurements. After

adjustment, all calibration slopes for runs 6 - 8 agree closely. This verifies that our sight glass data adjustment is appropriate. However, if the tank contained a liquid of unknown thermal expansion, another solution which we investigated was successful and is described below. After the last incremental volume transfer and all pertinent observations were obtained for runs 6 - 8, the sight glass was pressurized. The pressure used exceeded the tank head pressure, thereby forcing the liquid back inside the tank. When the pressure was removed, the liquid returned to the sight glass. Eight to ten such pressure cycles brought the sight glass and its scale much closer to the tank temperature as indicated by thermometry. At this time, the sight glass was observed again. A large change in the sight glass level was seen as shown below.

Sight Glass Observation

	Before Pressure Cycle	After Pressure Cycle	Tank Temp.
Run 6	294.22 cm @ 21.60 °C	293.75 cm @ 12.70 °C	10.52 °C
Run 7	280.80 cm @ 20.85 °C	280.42 cm @ 12.38 °C	10.75 °C
Run 8	285.81 cm @ 21.61 °C	285.44 cm @ 12.47 °C	11.10 °C

The merit of pressure cycling the sight glass may be judged by comparing the height difference between the initial and final liquid levels in the tank as measured by the XR-38 and the sight glass. The pressure cycled sight glass value is used for the final liquid-level measurement. The initial sight glass liquid is much closer to that of the tank liquid at that point. The height differences for each instrument are shown in the table below.

Calculated Height Difference

	Sight Glass	XR-38	S.G. Minus XR-38
Run 6	267.30 cm	267.37	- .07 cm
Run 7	250.70 cm	250.68	- .02 cm
Run 8	267.57 cm	267.58	.01 cm

The small differences between the measured heights for the two instruments shows clearly the effect of differing temperatures in the tank and sight glass and that the effect may be essentially eliminated by the use of pressure cycling the sight glass fluid. Also, this exercise shows clearly that differing temperatures between the sight glass and the tank fluids introduces a sizable systematic effect, especially for tall tanks.

When properly used, the sight glass performs to about the same precision as the other instruments tested and is free of serious bias.

Appendix 2. Density Probe Calibration and Use

In essence liquid density measured by head pressure is two determinations of liquid level from bubbler tubes (see fig. 5) at grossly different depths as explained later on. These pressure measurements occur at nearly the same time and liquid density is assumed constant during the period required.

The two observed head pressures, P_1 and P_2, are written as follows:

$$P_1 = \rho_L g L_1 \text{ and } P_2 = \rho_L g L_2$$

$$P_1 - P_2 = \rho_L g(L_1 - L_2)$$

where ρ_L is liquid density

L_1-L_2 is the difference in probe depth

g is the local acceleration of gravity

If the probe depth separation, L_1-L_2, is known, then ρ_L is calculated from the pressure measurements as follows:

$$\rho_L = \frac{P_1 - P_2}{g(L_1-L_2)} \qquad \text{eq. (1)}$$

The value for ρ_L is in kg/m^3 when

P_1 and P_2 are in pascals

g is meters/sec^2
(as measured at the site of calibration)

L_1 - L_2 is in meters.

The probe separation, L_1-L_2, can be determined by dimensional measurement and as reported here by head pressures in a liquid of known density. We feel the bubbler technique reflects true orifice behavior and deserves our attention since dimensional techniques are well known. When the bubbler tubes are assembled in place, as ours were, the glass tube of figure 3 can be replaced by a plastic bag. The bag must be supported externally to maintain a stable liquid level.

The calibration experiment follows the basic liquid level via head pressure scheme. Distilled water is the liquid of choice. It is readily available and the density is well characterized

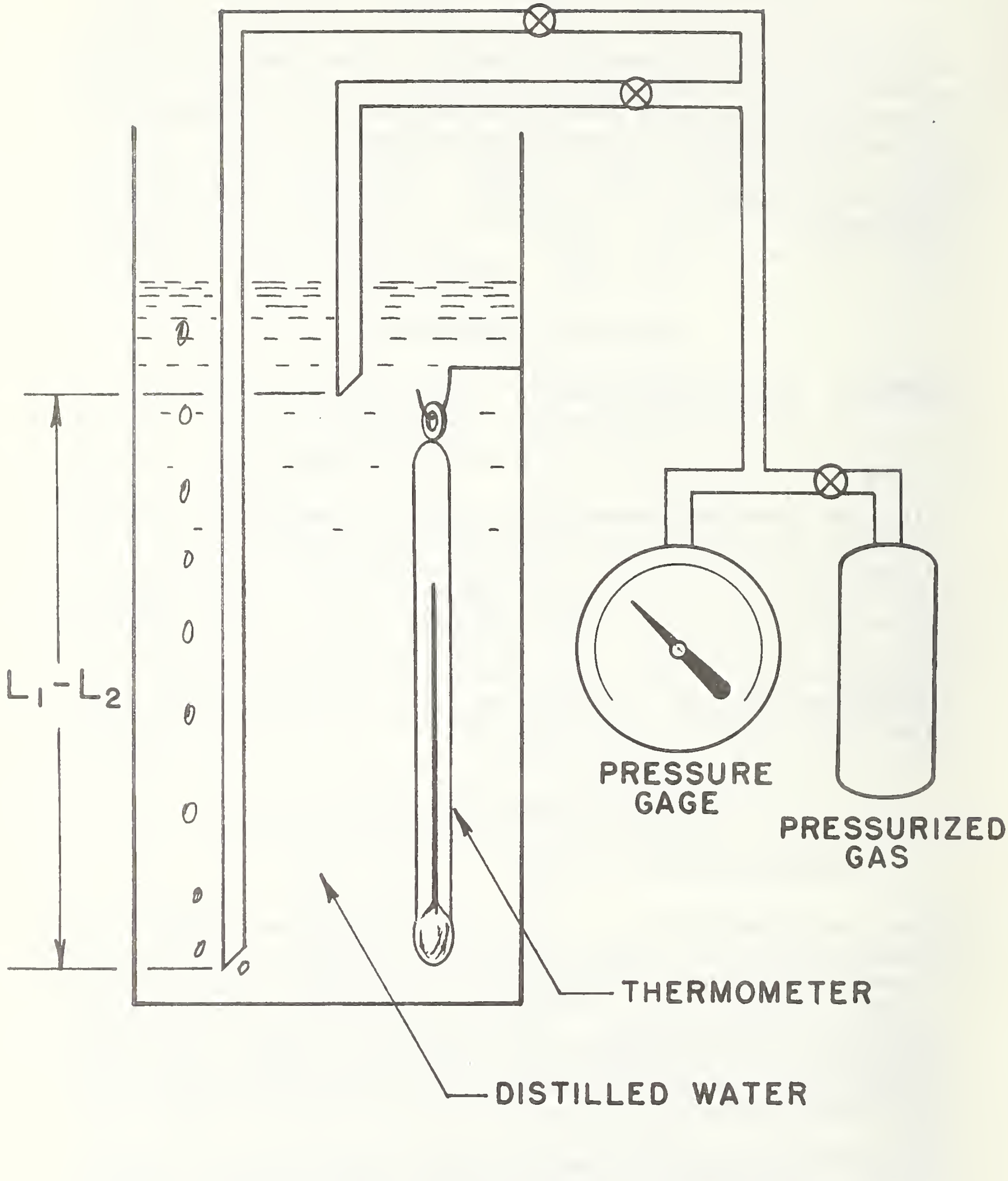

FIGURE 5: SIMPLIFIED VIEW OF THE CALIBRATION SCHEME FOR DETERMINING THE ORIFICE SEPARATION FOR USE IN THE LIQUID DENSITY MEASUREMENT.

from temperature measurement to 1 part in 10^{-5} or better although to maintain this level of accuracy, all components in contact with the distilled water must be extremely clean and nonsoluble in water.

Since the density of water is known sufficiently well, it follows from equation (1) that:

$$L_1 - L_2 = \frac{P_1 - P_2}{g\rho_L} \qquad \text{eq. (2)}$$

This value for L_1-L_2 can be substituted in equation (1) and unknown liquid densities can now be determined.

The temperature of distilled water must be measured to 0.01 °C for adequate accuracy of calculated densities. The recent work of Wagenbreth and Blanke [5] contains the necessary water density expansion formula with density expressed in cubic meters.

Liquid Density Measurement

During the calibration runs 2 - 8, the water density was determined after each volume increment was added to the tank by direct measurement. These determinations were obtained by observing the differential pressure between two probes using the XR-38. As described earlier, the pressure probes were installed in sets comprised of a level probe and one referred to as a density probe for each pressure gage. The additional pressure measurement required to determine density was only observed with the set connected to the XR-38 pressure gage.

Both probes were connected to each of the supporting braces closely adjacent to one another. The probe tips were tied together in a manner such that the orifice separation could not be changed.

Orifice separation was approximately 20 cm to simulate field use where the density measurement is made in the heel area of the tank. The upper probe was designated the density probe and the lower probe, the level detector. Occasionally the first volume increment failed to cover both probes causing a measurement ommission.

Although the data on measured densities are available for runs 2 - 8, we give only the data for run 2 in table 6. These data are typical of the other runs. Included in the table is a comparative analysis for runs 2 - 8. It is seen that the standard

RESIDUAL STANDARD DEVIATIONS (A)

Run No.	DENSITY (B) Calculated	DENSITY (B) Measured	HEIGHT (C) Calculated	HEIGHT (C) Measured
2	.0386	.517	.000527	.000974
3	.0270	.475	.000554	.000779
4	.0183	.545	.000270	.001111
5	.0296	.450	.000468	.000880
6	.0145	.351	.000525	.000877
7	.0066	.499	.000449	.001107
8	.0728	.424	.000592	.000903

(A) As successive increments of water were added, the temperature drifted downward and the density drifted upward. Residual standard deviations are computed from deviations from the trend line. A typical set of data, run 2, is shown below.

(B) Density in kilogram per cubic meter

(C) Height in meters

* *

NUMBER	VOLUME m^3	CAL. DEN. kg/m^3	CAL. HT. m	MEA. DEN. kg/m^3	MEA. HT. m
1	.189278	999.315132	.080646	.000000	.000000
2	.567973	999.433502	.416376	998.724243	.416672
3	.946503	999.477394	.749925	998.435822	.750709
4	1.325350	999.482384	1.084740	999.237724	1.085006
5	1.704280	999.488159	1.419691	999.021103	1.420355
6	2.082744	999.498245	1.754563	999.203156	1.755082
7	2.461034	999.515648	2.088275	997.714760	2.092049
8	2.839735	999.519005	2.421954	998.309845	2.424891
9	3.218276	999.526894	2.756424	998.532867	2.759171
	Intercept	999.373276	-0.085870	999.063629	-.086207
	Slope	.019932	.883291	-.075671	.884305
Std. Dev. of Ave.		.028082	.000353	.475361	.000827
Std. Dev. of Slope		.004990	.000180	.079783	.000397
Residual Std. Dev.		.038655	.000527	.517052	.000974

Table 6: Data from run 2 reduced and fitted using both calculated and measured density for comparison.

deviations of measured density and height are much larger than those for calculated density and height. This will result in a much larger calibration uncertainty, near the 1 part in 10^3 level. The error, ε_D, in our measured density has two components, ε_1 and ε_2, which are the random error of the XR-38 pressure gage and the systematic error of the assumed probe separation respectively.

Assuming the systematic component to be small although in our case it is not, the random component would occur twice for a given density determination. From table 5 we see that the XR-38 has a random component of 0.02 cm ($\hat{\sigma}_e$), therefore e_1 is estimated to be 0.040 cm for a probe separation of only 20 cm, about 2 parts in 10^3.

Obviously, since we calibrated the probe separation via the XR-38 pressure gage, there is also a sizable systematic error, ε_2. Of course, repeated calibration or another technique may reduce ε_2 to an acceptable magnitude.

For the random component effect on the density determination to be made comparable to that of the level detection, we need a probe separation of about 2 meters or more. This could be easily accomplished if a tank at least 2 meters long and 10 cm or more in diameter were placed inside a tank and used exclusively for the density measurement. In this way, the small tank could always be filled from the contents of the main tank after thorough mixing even when the liquid level is low.

Alternatively, the application of some other technique may be more suitable for measuring density.

APPENDIX 3

Gas Head Corrections for Level and Density Determinations

by

C.R. Tilford
Pressure and Vacuum Section
Heat Division

The non-zero density of the gas in the bubbler tubes used to determine liquid height and density gives rise to gas head cor- corrections to the simple equations generally used with these measurements. Under conceivable operating conditions, these corrections could well exceed the uncertainties reported for the measurements in this report and, in any case, restrict the operation of the system after it has been calibrated. Expressions for liquid height and density including gas head corrections will be derived and the effect of the corrections discussed.

These equations will be derived assuming that the gas density is linear with pressure (second order effects are at the parts per million level); the same gas is used in all parts of the system and its temperature is uniform throughout the system; and account is taken of effects due to bubble formation at the end of the probe tips. Effects due to nonlinearity or changes in calibration of the pressure gages are not considered.

Definitions:

The necessary quantities are defined below and illustrated in figure 6.

P_1, P_2, P_3, P_4	are the absolute pressures at the gages,
P_1', P_3', P_4'	are the absolute pressures at the probe tips,
P_2'	is the absolute pressure at the liquid surface,
h	is the vertical separation between probe tip #1 and the liquid surface,
ΔL	is the effective vertical separation between probe tips number 3 and 4 and is positive,
H_1	is the elevation of pressure gage #1 minus the elevation of probe tip #1,
H_2	is the elevation of pressure gage #2 minus the elevation of probe tip #3,
ρ	is the density of the liquid, and
ρ_1, ρ_2, ρ_3, ρ_4	are the gas densities at pressures P_1, P_2, P_3, and P_4.

Assuming that the density of gas is proportional to pressure, we have

$$\rho_i = AP_i \ (i = 1 \ . \ . \ . \ 4). \tag{1}$$

For air or N_2 at room temperature we have

$$A \approx 1.2 \times 10^{-8} \ g/cm^3 \ Pa$$

$$\approx 1.2 \times 10^{-3} \ g/cm^3 \ Atm.$$

Level Determination

We have

$$h = \frac{P_1' - P_2'}{g\rho}$$

where g is the local acceleration of gravity and ρ is the average density of the liquid. Taking gas heads between the appropriate points in or above the liquid and the gage #1 into account, we have

$$P_1' = P_1 + \rho_1 g H_1$$

$$P_2' = P_2 + \rho_2 g(H_1 - h)$$

From the above and using eq. (1) we have

$$g\rho h = P_1 + \rho_1 g H_1 - P_2 - \rho_2 g(H_1 - h)$$

$$= (P_1 - P_2)(1 + AgH_1) + \rho_2 gh$$

So

$$h = \frac{P_1 - P_2}{g(\rho - \rho_2)} (1 + AgH_1) \qquad (2)$$

Note that P_1-P_2 is the differential pressure at the gage; that if the tank is open to the atmosphere, P_2 is the atmospheric pressure at the gage and the correction ρ-ρ_2 will be about 0.12%, and that $Ag \approx 1.2 \times 10^{-4}$/m.

If gas density is ignored, we obtain the commonly used approximation to eq. (2)

$$h \approx \frac{P_1 - P_2}{g\rho} \qquad (2a)$$

The effect of the gas density corrections will depend on how the system is used. In many cases, including the measurements discussed in this report, what is really desired is not the height of the liquid but the volume of liquid, or more exactly, changes in the volume. The volume will be a function of h, or of $(P_1-P_2)/(\rho-\rho_2)$, which may be expressed as a polynomial. The coefficients of the polynomial can be determined by calibrating the tank with known volumes of liquid. If the same gas is used in both the calibration and operation and the relative elevation of gage #1 and probe tip #1 is not changed, the correction $(1 + AgH_1)$ will be the same for both calibration and operating conditions and will drop out. The volume can be expressed as

$$V = B + C\left[\frac{(P_1 - P_2)}{(\rho - \rho_2)}\frac{(\rho^* - \rho_2^*)}{(P_1^* - P_2^*)}\right] + D\left[\frac{(P_1 - P_2)}{(\rho - \rho_2)}\frac{(\rho^* - \rho_2^*)}{(P_1^* - P_2^*)}\right]^2 + \ldots .$$

where the quantities with an asterisk are those determined during the calibration. For a typical right circular cylindrical tank, such as is considered in this report, the linear term will be by far the largest.

If we consider this term we see that since the gas density is much smaller than the liquid density, we can approximate

$$\frac{\rho^* - \rho_2^*}{\rho - \rho_2} \approx \frac{\rho^*}{\rho}(1 - \rho_2^*/\rho)(1 + \rho_2/\rho)$$

$$\simeq \frac{\rho^*}{\rho}\left[1 + \frac{(\rho_2 - \rho_2^*)}{\rho} - \frac{\rho_2^*\rho_2}{\rho_2}\right]$$

The quadratic term is at the parts per million level and can be dropped. The linear term, $(\rho_2-\rho_2^*)/\rho$, will be zero if the gas composition, temperature and pressure at the top of the tank are the same during calibration and operation. Under these conditions, the approximate eq. (2a) can be used for both calibration and operation. If the tank is pressurized by, for example, 1 atmosphere after the tank has been calibrated, then the use of eq. (2a) would result in about a 0.12% error in the measured height or volume.

Similarly, if the relative elevation of the pressure gage and probe tip were to be changed by 8 m after the calibration an error, due to changes in the term $(1 + AgH_1)$, of about 0.1% would occur in the measured height or volume if the correction in eq. (2) was not applied.

It is hoped that this discussion will underscore the importance of not changing the system after it has been calibrated, or of making proper corrections if conditions are changed.

Density Determination

The density of the liquid in eqs. (2) or (2a) can be determined by measuring the differential pressure across a pair of bubblers separated by an effective vertical distance ΔL.

Thus

$$\rho = \frac{P_3' - P_4'}{g\Delta L}$$

Taking gas heads into account, we have

$$P_3' = P_3 + \rho_3 g H_2$$

$$P_4' = P_4 + \rho_4 g(H_2 - \Delta L)$$

Assuming that the same gas is used in both probes and using the above and eq. (1), we have

$$\rho g\Delta L = P_3 + \rho_3 g H_2 - P_4 - \rho_4 g(H_2 - \Delta L)$$

$$= (P_3 - P_4)(1 + AgH_2) + \rho_4\, g\Delta L$$

So

$$\rho = \frac{P_3 - P_4}{g\Delta L}(1 + AgH_2) + \rho_4 \tag{3}$$

Again, note that $(P_3 - P_4)$ is the measured differential pressure and an approximate form of eq. (3) is

$$\rho \approx \frac{(P_3 - P_4)}{g\Delta L} \tag{3a}$$

The correction term, ρ_4, occurs because the pressure generated by the column of liquid of height ΔL is partially offset by the pressure generated by the corresponding column of gas of density ρ_4. The correction $(1 + AgH_2)$, where $Ag \approx 1.2 \times 10^{-4}$/m for air or N_2, arises because the different pressures and correspondingly different gas densities in lines 3 and 4 give rise to different gas head corrections.

If eqs. (3) or (3a) are to be used with any pretext of accuracy, ΔL must be an effective length. This can be determined by making an in situ calibration of the entire density measuring system using a liquid of known density, ρ^*, such as water. From eq. (3) we then have

$$\Delta L = \frac{(P_3^* - P_4^*)}{\rho^* - \rho_4^*}(1 + AgH_2^*)$$

where ρ_4^* is the density of the gas at the calibration pressure P_4^*.

Then

$$\rho = \frac{(P_3 - P_4)(1 + AgH_2)(\rho^* - \rho_4^*)}{(P_3^* - P_4^*)(1 + AgH_2^*)} + \rho_4$$

If the relative elevation of gage #2 and the probe tips are not changed after calibration (H_2=H_2^*), we can write

$$\rho = \frac{(P_3 - P_4)}{(P_3^* - P_4^*)}(\rho^* - \rho_4^*) + \rho_4$$

$$= \frac{(P_3 - P_4)}{(P_3^* - P_4^*)}\rho^* + \rho_4 - \rho_4^* \frac{(P_3 - P_4)}{(P_3^* - P_4^*)}$$

$$\approx \frac{(P_3 - P_4)}{(P_3^* - P_4^*)}\rho^* + (\rho_4 - \rho_4^*)$$

since, if the densities of the calibration and measured liquids are approximately the same, $(P_3-P_4) \approx (P_3^*-P_4^*)$. Thus, if ρ_4=ρ_4^* we could use eq. (3a) for both the calibration and operation. Unfortunately, ρ_4 will change as the liquid level changes, or if the tank is pressurized. Obviously, the liquid level will change during operation and the error in using eq. (3a) could be significant for a large tank. As an example, if the liquid level differs by 8 m from the calibration level and eq. (3a) is used, the measured density will be in error by about 0.1%. The same sort of error will be introduced if the pressure above the liquid is changed by 1 atmosphere, or if the relative elevation of the gage and probe tips was changed by 8 m after calibration.

Thus, density measurements by the bubbler technique with uncertainties comparable to those obtained in the height measurements discussed in this report will not be possible unless the exact form of eq. (3) is used and care is taken in changing the operation of the system after it has been calibrated.

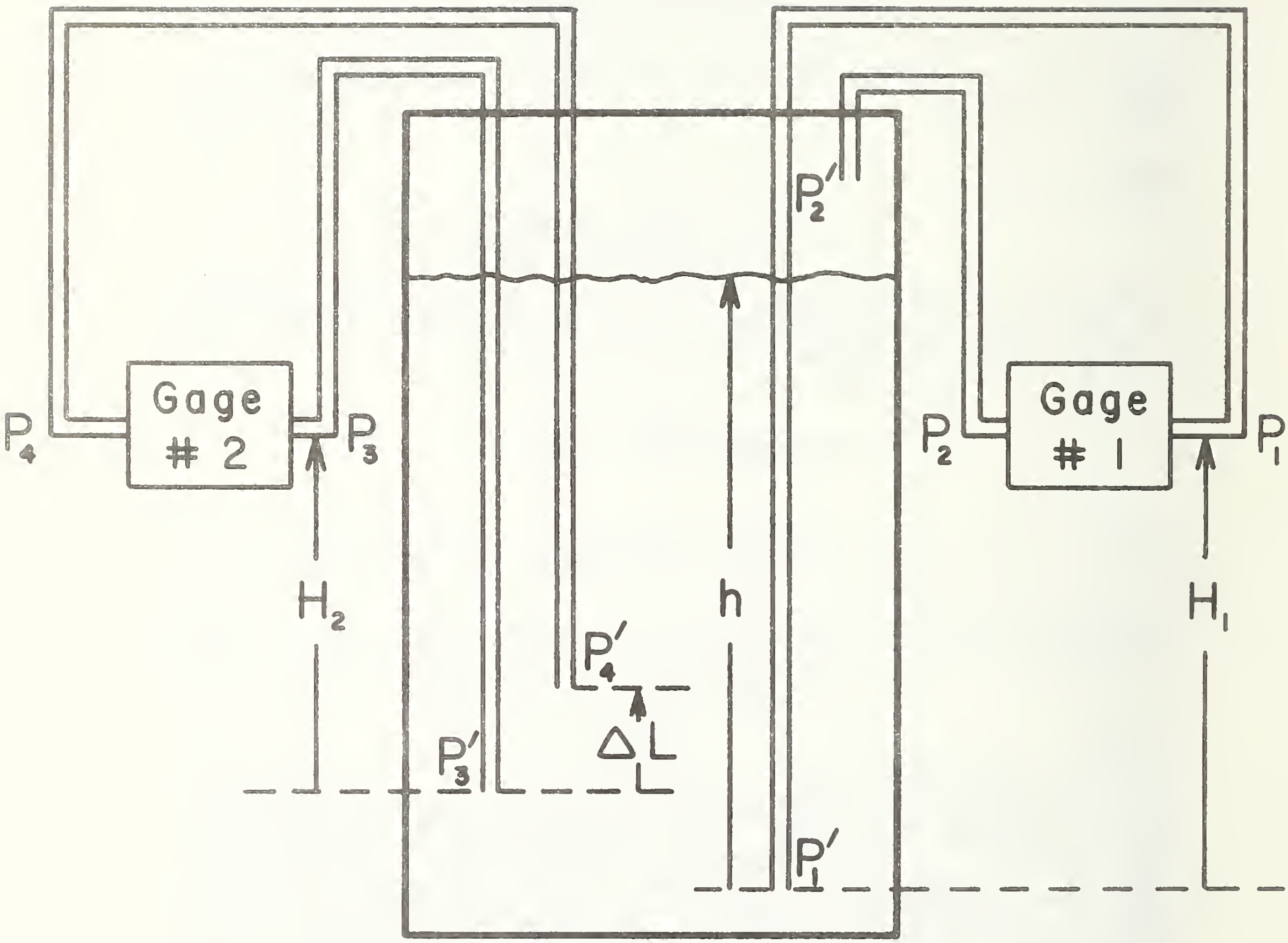

FIGURE 6: SCHEMATIC FOR PRESSURE DISCUSSION PRESENTED IN APPENDIX 3

NBS-114A (REV. 7-73)

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET	1. PUBLICATION OR REPORT NO. NBSIR 75-900	2. Gov't Accession No.	3. Recipient's Accession No.

4. TITLE AND SUBTITLE	5. Publication Date
Liquid Level Instrumentation in Volume Calibration	October 1975
	6. Performing Organization Code
7. AUTHOR(S) H.H. Ku, J.F. Houser, R.M. Schoonover, and J.R. Whetstone	8. Performing Organ. Report No. NBSIR 75-900
9. PERFORMING ORGANIZATION NAME AND ADDRESS NATIONAL BUREAU OF STANDARDS DEPARTMENT OF COMMERCE WASHINGTON, D.C. 20234	10. Project/Task/Work Unit No. 11. Contract/Grant No.
12. Sponsoring Organization Name and Complete Address (*Street, City, State, ZIP*) Sponsored in part by: Brookhaven National Laboratory Technical Support Organization Upton, L.I., New York	13. Type of Report & Period Covered Final 14. Sponsoring Agency Code

15. SUPPLEMENTARY NOTES

16. ABSTRACT (*A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.*)

The technique of pressure head measurement for determining liquid levels in tanks has been used for the intercomparison of commercially available electromanometers with themselves and a sight glass. In addition to obtaining intercomparison information, the high precision of these devices coupled with that of volumetric transfer techniques demonstrates the applicability of the calibration method in determining departures of the actual tank geometry from that of a model straight-walled geometry. It is shown that the calibration of the volume-to-height relationship of a 3300 liter tank may be accomplished with an uncertainty in this relationship of ~ 4 parts in 10,000.

17. KEY WORDS (*six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons*)

Key Words: Density measurement; electromanometer; large volume calibration.

18. AVAILABILITY	19. SECURITY CLASS (THIS REPORT)	21. NO. OF PAGES
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	20. SECURITY CLASS (THIS PAGE) UNCLASSIFIED	22. Price $4.00

USCOMM-DC 29042-P74